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INFLUENCE OF SHRINKAGE STRESSES ON THE STABILITY OF COMPOSITE LAMINATED MATERIALS

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16. Abstract

The mechanism of stability loss in the microvolume of laminated composites during shrinkage is studied on the basis of three-dimensional linearized stability equations at small subcritical deformations. It is assumed that the layers after having been combined to a monolith at a certain temperature are subjected to cooling. It is shown that instability will always occur in the shortwave mode for composites with small filler concentrations.

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A. N. Guz'

The study [4] established experimentally that during the /11*
shrinkage of laminated materials consisting of materials whose physico-mechanical properties differ greatly (glass-plastics and metal-plastics) stability losses occur in the micro-volume, which has a great influence on the stability of the laminated material. This was the basis for studies of compressive strength of ** materials [1, 4 — 6], based on different approximate /12
methods for determining the interaction between the filler and the binder.

Let us examine the mechanism for stability loss in the micro-volume of a laminated composite material during shrinkage, using three-dimensional linearized stability equations for small subcritical deformations. The laminated material consists of layers which alternate in the direction of the Ox_2 axis. We shall assume that the layers are infinite in the direction of the Ox_3 axis (see the figure), the layers of the binder and the filler are isotropic and have the following parameters: λ, μ, E, α_T, ν and $\lambda^{(1)}$, $\mu^{(1)}$, $E^{(1)}$, $\alpha_T^{(1)}$, (α_T — volumetric expansion coefficient). We shall use the index "1" to designate all of the quantities referring to the filler. Each layer is referred to

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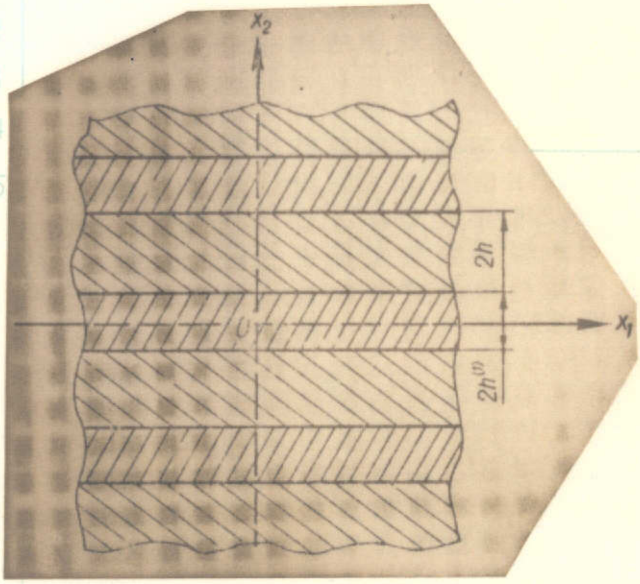
*Numbers in the margin indicate pagination of foreign text.

**[Translator's Note: illegible in foreign text.]



the system of coordinates x_{1i}, x_{2i}, x_{3i} и $x_{1i}^{(1)}$, $x_{2i}^{(1)}$, $x_{3i}^{(1)}$, which are obtained from the coordinate system x_1, x_2, x_3 (see the figure) by parallel transposition along the Ox_2 axis, and are connected with the middle planes of the corresponding layers.

We shall assume that the layers are combined in a monolith at the temperature T, and are then cooled. Let us study the possibility of stability losses in a micro-volume at a temperature $T_{av} < T$ under conditions of plane deformation. The change /13
in the physico-mechanical properties of the binder during the cooling process will not be considered. This assumption keeps the stability in reserve, since it corresponds to an increase in the compression stresses. Let us determine the components of the subcritical state, designating their index by zero.

$$\left.\begin{aligned} \sigma_{11}^{0} &= (\lambda+2\mu)\varepsilon_{11}^{0} + \lambda\varepsilon_{22}^{0} + (3\lambda+2\mu)\alpha_T\Delta T; \\ \sigma_{22}^{0} &= (\lambda+2\mu)\varepsilon_{22}^{0} + \lambda\varepsilon_{11}^{0} + (3\lambda+2\mu)\alpha_T\Delta T; \\ \Delta T &= T - T_{cr}; \quad \sigma_{12}^{0} = 0. \end{aligned}\right\} \qquad (1)$$

It is necessary to substitute the index "1" for the filler in all the quantities of (1), except ΔT. We obtain from the element equilibrium condition

$$\sigma_{11}^{0} h + \sigma_{11}^{(1)0} h^{(1)} = 0. \qquad (2)$$

It may be stated that if ε_{11}^{0}, $\varepsilon_{11}^{(1)0}$, ε_{22}^{0} and $\varepsilon_{22}^{(1)0}$ are considered to be constant values and we set $\sigma_{22}^{0} = \sigma_{22}^{(1)0} = 0$, the conditions for the compression of the layers will be satisfied if

$$\varepsilon_{11}^{0} = \varepsilon_{11}^{(1)0}. \tag{3}$$

Thus

$$\begin{gathered}
\sigma_{11}^{0} = \left(\lambda + 2\mu - \frac{\lambda^2}{\lambda + 2\mu}\right)\varepsilon_{11}^{0} + 2(3\lambda + 2\mu)\frac{\mu}{\lambda + 2\mu}\alpha_T \Delta T; \\
\sigma_{11}^{(1)0} = \left(\lambda^{(1)} + 2\mu^{(1)} - \frac{\lambda^{(1)2}}{\lambda^{(1)} + 2\mu^{(1)}}\right)\varepsilon_{11}^{0} + 2(3\lambda^{(1)} + 2\mu^{(1)})\frac{\mu}{\lambda^{(1)} + 2\mu^{(1)}}\alpha_T \Delta T.
\end{gathered} \tag{4}$$

We may determine ε_{11}^{0} from (2) and (4)

$$\begin{gathered}
\varepsilon_{11}^{0} = -\frac{1}{2}\Delta T[\mu(3\lambda + 2\mu)(\lambda^{(1)} + 2\mu^{(1)})\alpha_T h + \mu^{(1)}(3\lambda^{(1)} + 2\mu^{(1)})(\lambda + \\
+ 2\mu)\alpha_T^{(1)} h^{(1)}][(\lambda + \mu)(\lambda^{(1)} + 2\mu^{(1)})\mu h + (\lambda^{(1)} + \mu^{(1)})(\lambda + 2\mu) h^{(1)} \mu^{(1)}]^{-1};
\end{gathered} \tag{5}$$

Thus, the subcritical stress state is characterized by the following components which differ from zero

$$\sigma_{11}^{(1)0} = -p^{(1)}; \quad \sigma_{11}^{0} = -p; \quad p = -\frac{h^{(1)}}{h} p^{(1)};$$

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$$\begin{gathered}
p^{(1)} = 2h\Delta T \mu \mu^{(1)}[(\lambda^{(1)} + \mu^{(1)})(3\lambda + 2\mu)\alpha_T - (\lambda + \mu)(3\lambda^{(1)} + 2\mu^{(1)})\alpha_T^{(1)}] \times \\
\times [\mu h(\lambda + \mu)(\lambda^{(1)} + 2\mu^{(1)}) + \mu^{(1)} h^{(1)}(\lambda^{(1)} + \mu^{(1)})(\lambda + 2\mu)]^{-1}.
\end{gathered} \tag{6}$$

To determine the perturbations, we shall use three-dimensional linearized equations for small subcritical deformations [3]. We may represent the components of the perturbations of the stress and deformed states for the $l^{\underline{th}}$ layer of the binder, according to [2], in the form

$$\left.\begin{aligned}
&u_{1i}=-(\lambda+\mu)\frac{\partial^2}{\partial x_{1i}\partial x_{2i}}\psi_i; \quad u_{2i}=\left[(\lambda+2\mu)\frac{\partial^2}{\partial \xi_{1i}^2}+\mu\frac{\partial^2}{\partial x_{2i}^2}\right]\psi_i;\\
&\sigma_{11i}=\lambda\mu\left(\frac{\partial^2}{\partial x_{2i}^2}-\frac{\lambda+2\mu}{\lambda}\cdot\frac{\partial^2}{\partial x_{1i}^2}\right)\frac{\partial}{\partial x_{2i}}\psi_i;\\
&\sigma_{22i}=(\lambda+2\mu)\mu\left(\frac{\partial^2}{\partial x_{2i}^2}+\frac{(\lambda+2\mu)^2-\lambda^2-\lambda\mu}{\mu(\lambda+2\mu)}\cdot\frac{\partial^2}{\partial x_{1i}^2}\right)\frac{\partial}{\partial x_{2i}}\psi_i;\\
&\sigma_{12i}=-\lambda\mu\left(\frac{\partial^2}{\partial x_{2i}^2}-\frac{\lambda+2\mu}{\lambda}\cdot\frac{\partial^2}{\partial x_{1i}^2}\right)\frac{\partial}{\partial x_{1i}}\psi_i.
\end{aligned}\right\} \tag{7}$$

The functions ψ_i are determined as follows:

$$\psi_i=\psi_{i,1}+\psi_{i,2}; \quad \left(\frac{\partial^2}{\partial x_{2i}^2}+\zeta_j^2\frac{\partial^2}{\partial x_{1i}^2}\right)\psi_{i,j}=0. \tag{8}$$

The parameters $\zeta_{1,2}^2$ are determined from the formulas

$$\zeta_{1,2}^2=1-\frac{1}{2}\cdot\frac{P}{\mu}\cdot\frac{\mu}{\lambda+2\mu}\pm\sqrt{\frac{P}{\mu}\cdot\frac{\lambda+\mu}{\lambda+2\mu}+\left(\frac{1}{2}\cdot\frac{P}{\mu}\cdot\frac{\mu}{\lambda+2\mu}\right)^2}. \tag{9}$$

To determine similar values in the filler, it is necessary to substitute the index "I" in Formulas (7) — (9) in all of the quantities.

The combining conditions on the surfaces of the binder and filler may be written in the usual form.

Let us consider two forms of stability loss. The first form is symmetrical when the layers of the binder and the filler lose stability in terms of identical forms and in one phase. The second form is asymmetrical, when the layers of the binder and the filler lose stability in terms of identical forms but in the opposite phase. /15

For the first form, we may select the solution in the form

$$\varphi_i = \left(A \operatorname{ch} \frac{\pi}{l} \zeta_1 x_{2i} + B \operatorname{ch} \frac{\pi}{l} \zeta_2 x_{2i}\right) \sin \frac{\pi}{l} x_1 ;$$
$$\varphi_i^{(1)} = \left(A^{(1)} \operatorname{ch} \frac{\pi}{l} \zeta_1^{(1)} x_{2i}^{(1)} + B^{(1)} \operatorname{ch} \frac{\pi}{l} \zeta_2^{(1)} x_{2i}^{(1)}\right) \sin \frac{\pi}{l} x_1 . \quad (10)$$

The solution in the form (10) satisfies the symmetry and periodicity conditions with respect to x_2, if the combining conditions are only satisfied when

$$x_{2i} = h , \quad x_{2i}^{(1)} = -h^{(1)}$$

As a result of the regular procedure, we derive the characteristic equation in the following form:

$$\det \| \beta_{ij} \| = 0, \quad i,j = 1,2,3,4. \quad (11)$$

Here

$$\beta_{11}(\zeta_1) = [(\lambda + 2\mu)\mu \zeta_1^2 - \mu(3\lambda + 4\mu)] \zeta_1 \operatorname{sh} \alpha \zeta_1 ; \quad \beta_{12} = \beta_{11}(\zeta_2) ;$$

$$\beta_{13}(\zeta_1^{(1)}) = [(\lambda^{(1)} + 2\mu^{(1)})\mu^{(1)} \zeta^{(1)2} - \mu^{(1)}(3\lambda^{(1)} + 4\mu^{(1)})] \zeta_1^{(1)} \operatorname{sh} \alpha^{(1)} \zeta_1^{(1)} ,$$

$$\beta_{14} = \beta_{13}(\zeta_2^{(1)}) ; \quad \beta_{21}(\zeta_1) = \mu(\lambda \zeta_1^2 + \lambda + 2\mu) \operatorname{ch} \alpha \zeta_1 ; \quad \beta_{22} = \beta_{21}(\zeta_2) ;$$

$$\beta_{23}(\zeta_1^{(1)}) = -\mu^{(1)}(\lambda^{(1)} \zeta_1^{(1)2} + \lambda^{(1)} + 2\mu^{(1)}) \operatorname{ch} \alpha^{(1)} \zeta_1^{(1)} ; \quad \beta_{24} = \beta_{23}(\zeta_2^{(1)}) ;$$

$$\beta_{31}(\zeta_1) = (\mu \zeta_1^2 - \lambda - 2\mu) \operatorname{ch} \alpha \zeta_1 ; \quad \beta_{32} = \beta_{31}(\zeta_2) ; \quad \beta_{33}(\zeta_1^{(1)}) = -$$

$$-(\mu^{(1)} \zeta_1^{(1)2} - \lambda^{(1)} - 2\mu^{(1)}) \operatorname{ch} \alpha^{(1)} \zeta_1^{(1)} ; \quad \beta_{34} = \beta_{33}(\zeta_2^{(1)}) ; \quad \beta_{41}(\zeta_1) =$$

$$= (\lambda + \mu) \zeta_1 \operatorname{sh} \alpha \zeta_1 ; \quad \beta_{42} = \beta_{41}(\zeta_2) ; \quad \beta_{43}(\zeta_1^{(1)}) = (\lambda^{(1)} + \mu^{(1)}) \zeta_1^{(1)} \operatorname{sh} \alpha^{(1)} \zeta_1^{(1)} ;$$

$$\beta_{44} = \beta_{43}(\zeta_2^{(1)}), \text{ где } \alpha = \pi \frac{h}{l} , \ \alpha^{(1)} = \pi \frac{h^{(1)}}{l} .$$

For the second form of stability loss, we may write the /16
solution in the form

$$\begin{aligned}\psi_i &= \left(A\,sh\frac{\pi}{l}\zeta_1 x_{2i} + B\,sh\frac{\pi}{l}\zeta_2 x_{2i}\right)\sin\frac{\pi}{l}x_{1i};\\ \psi_i^{(1)} &= \left(A^{(1)}ch\frac{\pi}{l}\zeta_1^{(1)}x_{2i}^{(1)} + B^{(1)}ch\frac{\pi}{l}\zeta_2^{(1)}x_{2i}^{(1)}\right)\sin\frac{\pi}{l}x_{1i}.\end{aligned} \tag{13}$$

These solutions are written for the $i^{\underline{th}}$ layer of the binder and the $i^{\underline{th}}$ layer of the filler. In order to continue the solutions (13) peridically with respect to $0x_2$, the solutions for the $i + 1^{\underline{th}}$ layer and the $i - 1^{\underline{th}}$ layer of the filler and the binder must be selected in the form (13), except that the constants A, B, $A^{(1)}$ and $B^{(1)}$ must be taken with the opposite sign. Substituting the solution in the combining condition, we obtain the characteristic equation in the form (11), whose elements have the form (12), if $ch\alpha\zeta_i$ and $sh\alpha\zeta_i$ change places in the latter.

As a result of the numerical solution of (11), we obtain the dependence $p=f(l)$, the critical value of P, and consequently, ΔT is determined as a result of minimizing $P_{cr}=\min f(l)$.

Let us examine the particular case when the following inequalities are satisfied

$$h \gg h^{(1)};\quad \alpha \gg 1;\quad \alpha^{(1)} \ll 1, \tag{14}$$

i.e., when the layers of the binder are much thicker than the filler layers, the long-wave mode of the stability losses is much less than the thickness of the binder layer and much greater than the thickness of the filler layer. To determine the first term of the expansion of the root, taking into account (4), we obtain the equation

$$\zeta_1\zeta_2(\zeta_1^2-\zeta_2^2)(\zeta_1^{(1)^2}-\zeta_2^{(1)^2})=0. \tag{15}$$

For real materials, we may write the inequalities

$$\alpha_T > \alpha_T^{(1)}; \quad E < E^{(1)}. \tag{16}$$

We obtain the following from (6) and (16)

$$p^{(1)} > 0; \quad p < 0. \tag{17}$$

From (15) and relationships (6) and (17), we find that equation

$$\zeta_1^{(1)2} - \zeta_2^{(1)2} = 0$$

has no roots. Two other equations have only one root /17

$$p^{(1)} = 4\frac{h}{h^{(1)}}\mu\frac{(\lambda+2\mu)(\lambda+\mu)}{\mu^2}, \tag{18}$$

which has no physical meaning for real materials.

Thus, we reach the conclusion that, for composite laminated materials with a small filler concentration, stability loss is impossible for the long-wave mode. The experimental studies [4] did not observe stability loss in the long-wave mode either, but the stability loss took place for the shortwave mode.

A change in the physico-mechanical properties of the binder in the case of shrinkage can be taken into account by dividing the entire interval into several intervals and assuming that the physico-mechanical properties do not depend on temperature in each of the intervals.

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